DIRECT AND INVERSE VARIATION

This unit is about direct and inverse relationships. Direct variation is a linear function defined by an equation of the form $y = kx$ when x is not equal to zero. Inverse variation is a nonlinear function defined by an equation of the form $xy = k$ when x is not equal to zero and k is a nonzero real number constant.

Direct Variation

Inverse Variation

Direct Variation

The variable y varies directly as x if there is a nonzero constant k such that $y = kx$. The equation $\boldsymbol{y = kx}$ is called a direct variation equation and the number k is called the constant of variation.

*There are many situations in which one quantity varies directly as another:

- an employee's wages vary directly to the number of hours worked
- the amount of sales tax varies directly to the total price of the merchandise

y varies directly to x

To find the constant of variation (k) and the direct-variation equation, use the following steps.

1. Replace the x and y with the given values.
2. Solve for k.
3. Replace k in the direct variation equation.

Example 1: Find the constant of variation (k), and the direct-variation equation, if y varies directly as x and $y = -72$ when $x = -18$.

Step #1: Replace x and y with the given values.

$$y = kx$$

$$-72 = k(-18) \qquad y = -72, \qquad x = -18$$

Step #2: Solve for k.

$$\frac{-72}{-18} = \frac{k\,(\cancel{-18})}{\cancel{-18}} \qquad \text{Divide both sides by } -18.$$

$$4 = k$$

Step #3: Replace k in the direct variation equation.

$$y = kx$$

$$y = 4x$$

Example 2: Each day Michael roller blades for exercise. When traveling at a constant rate, he travels 4 miles in about 20 minutes. At this rate, how long will it take Michael to travel seven miles?

To solve:

First, find a direct variation equation that models Michael's distance as it varies with time using $d = rt$.

Distance (d) varies directly as (t) and rate (r) is the constant of variation.

$$y = kx \rightarrow \quad d = rt$$

Step 1: Find the constant of variation (r).

$$d = rt$$

$$4 \text{ miles} = r(20 \text{ minutes}) \qquad d = 4 \text{ miles}, \; t = 20 \text{ minutes}$$

$$\frac{4 \text{ mi}}{20 \text{ min}} = \frac{r\,(\cancel{20 \text{ min}})}{\cancel{20 \text{ min}}}$$

Divide both sides by 20 minutes.

$$r = \frac{4 \text{ mi}}{20 \text{ min}} = \frac{1 \text{ mi}}{5 \text{ min}} \text{ or } \frac{1}{5} \text{ mile per minute}$$

Step 2: Write the direct variation equation.

$$d = rt \qquad r = \frac{1}{5}$$

$$d = \frac{1}{5}t$$

Now, use the direct variation equation to solve the problem.

Step 3: Apply the direct variation equation.

$d = \frac{1}{5}t$

$7 = \frac{1}{5}t$ Substitution ($d = 7$ miles)

$(5)7 = \cancel{(5)}\frac{1}{\cancel{5}}t$ Multiply both sides by 5.

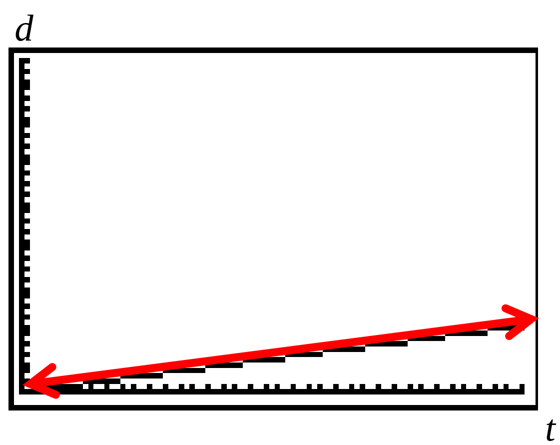
$35 = t$

Thus, at the rate of 4 miles in 20 minutes, it will take Michael 35 minutes to travel seven miles.

The function above is graphed below. Study the graph carefully. As ***t* increases, *d* increases**.

For example:

When t is 5, d is 1.
When t increases to 10, d increases to 2.
When t increases to 35, d increases to 7.



*Note: the graph is **linear**.

*This graph was created on a graphing calculator, causing pixelation of the straight line. Thus, a **straight red line** was added to show the true appearance of the linear graph.

Inverse Variation

An inverse variation is a function that is defined by an equation in the following form: $xy = k$ where k is a nonzero real-number constant.

*There are many situations in which one quantity varies indirectly as another:

- as the rate increases, the time decreases when traveling a set distance.
- the volume of a gas in a container decreases as the pressure increases and the temperature remains constant.

Consider the following expressions:

$$xy = k$$

$$\frac{\cancel{x}y}{\cancel{x}} = \frac{k}{x} \qquad \text{Divide both sides by } x.$$

$$y = \frac{k}{x}$$

$$y = k \cdot \frac{1}{x} \qquad \text{Another way to write } \frac{k}{x} \text{ is } k \cdot \frac{1}{x}.$$

Thus, y is directly proportional to the multiplicative inverse of x, $\frac{1}{x}$.

y* varies inversely to *x

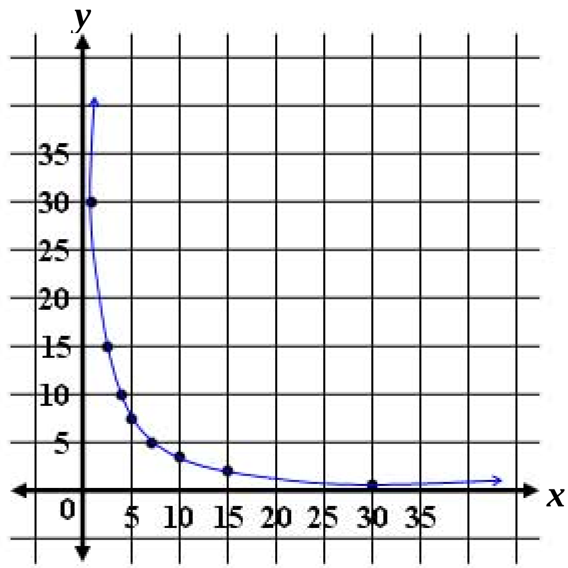
Example: Examine the following function: $xy = 30 \ (k = 30)$. What are some values for *y* that correspond to *x*?

The given function is graphed below. Study the graph carefully. As ***x* increases, *y* decreases**.

$$xy = 30 \quad \rightarrow \quad y = \frac{30}{x} \quad \rightarrow \quad y = \frac{1}{x} \cdot 30$$

For example:

When *x* is 1, *y* is 30…(1, 30) .
When *x* increases to 2, *y* decreases to 15…(2, 15).
When *x* increases to 10, *y* decreases to 3…(3, 10).
When *x* increases to 20, *y* decreases to 1 1/2…(20, 1.5).



*Note: the graph is **nonlinear**.