POLYNOMIAL OPERATIONS

ADDITION AND SUBTRACTION:

Adding and subtracting polynomials is the same as the procedure used in combining like terms. When *adding* polynomials, simply drop the parenthesis and combine like terms. When *subtracting* polynomials, distribute the negative first, then combine like terms.

Examples:

Addition:

$$(2x^2+3x-7)+(3x^2-4x-10)=2x^2+3x^2+3x-4x-7-10=5x^2-x-17$$

Subtraction:

$$(5x^2-12x+1)-(2x^2+3x-7)=5x^2-12x+1-2x^2-3x+7=3x^2-15x+8$$

MULTIPLICATION:

1. Monomial times Monomial: To multiply a monomial times a monomial, just multiply the numbers then multiply the variables using the rules for exponents.

   Example:

   $$(-2x^2y)(5xy^7)=-2\cdot 5x^2\cdot x\cdot y\cdot y^7=-10x^3y^8$$

2. Monomial times Polynomial: Simply use the distributive property to multiply a monomial times a polynomial.

   Examples:

   a. $-2x(x^2+3x-8)=-2x(x^2)-2x(3x)-2x(-8)=-2x^3+6x^2+16x$

   b. $5x^2(-2x^4+3y-6)=5x^2(-2x^4)+5x^2(3y)+5x^2(-6)=-10x^6+15x^2y-30x^2$

3. Binomial times a Binomial: To multiply two binomials, use the **FOIL** method (**F**irst times first, **O**utside times outside, **I**nside times inside, and **L**ast times last).

   Example:

   $$(x+2)(x-3)=x(x)+x(-3)+2(x)+2(-3)=x^2-3x+2x-6=x^2-x-6$$

   Special Products: The following formulas may be used in these special cases as a short cut to the FOIL method.

   Difference of Squares:
   $(a+b)(a-b)=a^2-b^2$

   Example:
   $(3x+4)(3x-4)=9x^2-16$

   Perfect Squares:
   $(a+b)^2=a^2+2ab+b^2$

   Example:
   $(x+4)^2=x^2+2(x)(4)+4^2=x^2+8x+16$

   $(a-b)^2=a^2-2ab+b^2$

   Example:
   $(x-3)^2=x^2-2(x)(3)+3^2=x^2-6x+9$

4. Polynomial times polynomial: To multiply two polynomials where at least one has more than two terms, distribute each term in the first polynomial to each term in the second.

   Examples:

   a. $(x^2 + 3x - 4)(x^2 - 6x + 5)$
   $$= x^2(x^2) + x^2(-6x) + x^2(5) + 3x(x^2) + 3x(-6x) + 3x(5) - 4(x^2) - 4(-6x) - 4(5)$$
   $$= x^4 - 6x^3 + 5x^2 + 3x^3 - 18x^2 + 15x - 4x^2 + 24x - 20$$
   $$= x^4 - 3x^3 - 17x^2 + 39x - 20$$

   b. $(2x - 3)(4x^2 - 5x + 1)$
   $$= 2x(4x^2) + 2x(-5x) + 2x(1) - 3(4x^2) - 3(-5x) - 3(1)$$
   $$= 8x^3 - 10x^2 + 2x - 12x^2 + 15x - 3$$
   $$= 8x^3 + 22x^2 + 17x - 3$$

DIVISION:

1. Division by Monomial: Each term of the polynomial is divided by the monomial and it is simplified as individual fractions.

   Examples:

   a. $\frac{3x^2-9x+14}{3x} = \frac{3x^2}{3x} - \frac{9x}{3x} + \frac{14}{3x} = x - 3 + \frac{14}{3x}$

   b. $\frac{10y^2-25y+20}{-5y} = \frac{10y^2}{-5y} - \frac{25y}{-5y} + \frac{20}{-5y} = -2y + 5 - \frac{4}{y}$

2. Division by Binomial or Larger Polynomial:

   Use the long division format as follows:

   - Both the divisor and the dividend must be written in descending order.
   - Any missing powers should be replaced by zero.
   - All remainders are in fraction form $(\text{remainder}/\text{divisor})$ and are added to the quotient.

   Examples:

   a. $(x^2 - 2x - 15) \div (x + 3) = x - 5$

   $$\begin{array}{r} x-5 \\ x+3\,\overline{)\,x^2-2x-15} \\ \underline{-(x^2+3x)} \\ -5x-15 \\ \underline{-(-5x-15)} \\ 0 \end{array}$$

   b. $\frac{9x^3-x+3}{3x-2} = 3x^2 + 2x + 1 + \frac{5}{3x-2}$

   $$\begin{array}{r} 3x^2+2x+1 \\ 3x-2\,\overline{)\,9x^3+0x^2-x+3} \\ \underline{-(9x^3-6x^2)} \\ 6x^2-x \\ \underline{-(6x^2-4x)} \\ 3x+3 \\ \underline{-(3x-2)} \\ 5 \end{array}$$