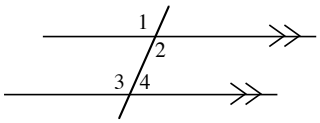
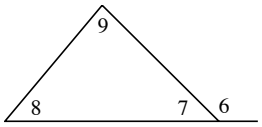
PROPERTIES OF ANGLES, LINES, AND TRIANGLES #2

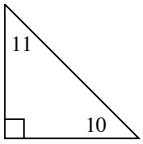
Parallel lines



- corresponding angles are equal: $m\angle 1 = m\angle 3$
- alternate interior angles are equal: $m\angle 2 = m\angle 3$
- $m\angle 2 + m\angle 4 = 180^\circ$

Triangles



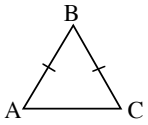


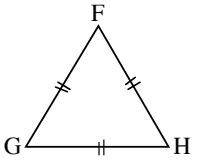
- $m\angle 7 + m\angle 8 + m\angle 9 = 180^\circ$
- $m\angle 6 = m\angle 8 + m\angle 9$ (exterior angle = sum remote interior angles)
- $m\angle 10 + m\angle 11 = 90^\circ$ (complementary angles)

Also shown in the above figures:

- vertical angles are equal: $m\angle 1 = m\angle 2$
- linear pairs are supplementary: $m\angle 3 + m\angle 4 = 180^\circ$ and $m\angle 6 + m\angle 7 = 180^\circ$

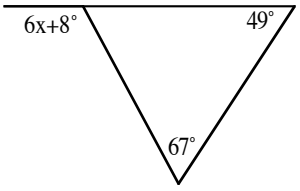
In addition, an isosceles triangle, ΔABC, has $BA=BC$ and $m\angle A = m\angle C$. An equilateral triangle, ΔGFH, has $GF = FH = HG$ and $m\angle G = m\angle F = m\angle H = 60^\circ$.





Example 1

Solve for x.

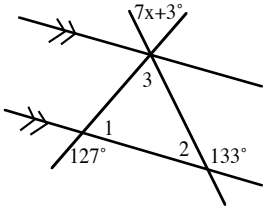


Use the Exterior Angle Theorem: $6x + 8^\circ = 49^\circ + 67^\circ$

$6x^\circ = 108^\circ \Rightarrow x = \frac{108^\circ}{6} \Rightarrow x = 18^\circ$

Example 2

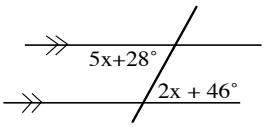
Solve for x.



There are a number of relationships in this diagram. First, $\angle 1$ and the 127° angle are supplementary, so we know that $m\angle 1 + 127^\circ = 180^\circ$ so $m\angle 1 = 53^\circ$. Using the same idea, $m\angle 2 = 47^\circ$. Next, $m\angle 3 + 53^\circ + 47^\circ = 180^\circ$, so $m\angle 3 = 80^\circ$.

Because angle 3 forms a vertical pair with the angle marked $7x + 3^\circ$, $80^\circ = 7x + 3^\circ$, so $x = 11^\circ$.

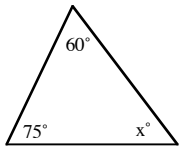
Example 3

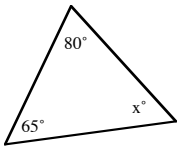


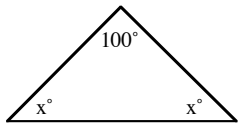
Find the measure of the acute alternate interior angles.

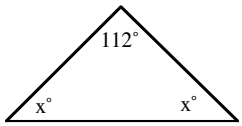
Parallel lines mean that alternate interior angles are equal, so $5x + 28^\circ = 2x + 46^\circ \Rightarrow 3x = 18^\circ \Rightarrow x = 6^\circ$. Use either algebraic angle measure: $2(6^\circ) + 46^\circ = 58^\circ$ for the measure of the acute angle.

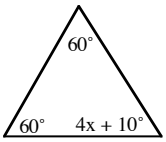
Use the geometric properties and theorems you have learned to solve for x in each diagram and write the property or theorem you use in each case.

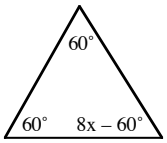
1. 

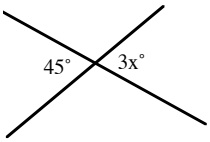
2. 

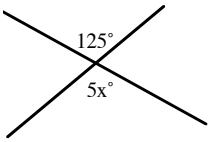
3. 

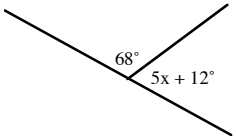
4. 

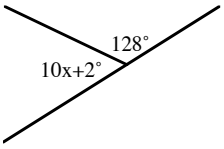
5. 

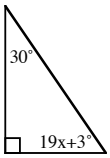
6. 

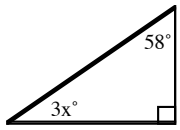
7. 

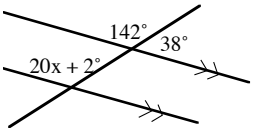
8. 

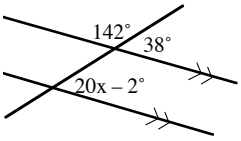
9. 

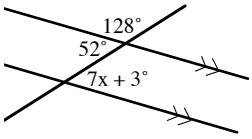
10. 

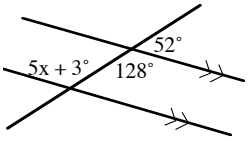
11. 

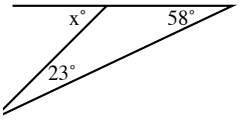
12. 

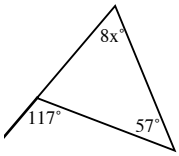
13. 

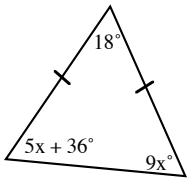
14. 

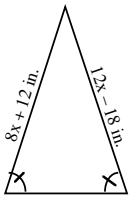
15. 

16. 

17. 

18. 

19. 

20. 

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